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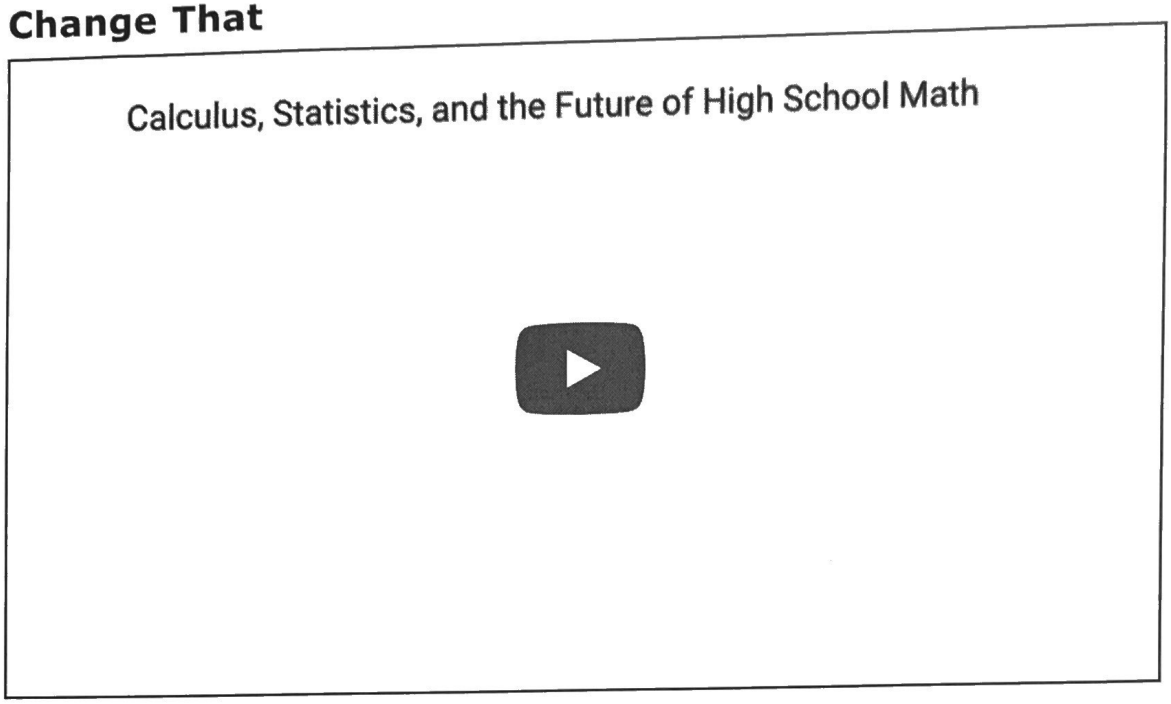
Calculus Is the Peak of High School Math. Maybe It's Time to Change That



By Sarah D. Sparks

May 22, 2018

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For more than 30 years, calculus has been seen as the pinnacle of high school math—essential for careers in the hard sciences, and an explicit or unspoken prerequisite for top-tier colleges.

But now, **math and science professionals are beginning to question** how helpful current high school calculus courses really are for advanced science fields. The ubiquitous use of data in everything from physics and finance to politics and education is helping to build momentum for a new path in high school math—one emphasizing statistics and data literacy over calculus.

"We increasingly understand the world around us through data: gene expression, identifying new planets in distant solar systems, and everything in between," said Randy Kochevar, a senior research scientist at the Education Development Center, an international nonprofit that works with education officials. Statistics and data analysis, he said, "is fundamental to many of the things we do routinely, not just as scientists but as professionals."

He and other experts are still debating the best way to integrate a new approach in an already crowded high school curriculum. One of the most difficult philosophical challenges: how to prevent a statistics path from replicating the severe tracking and equity problems that have long existed in classical mathematics.

"There's a sense that calculus is up here and statistics is a step below," said Dan Chase, a secondary mathematics teacher at Carolina Day School in North Carolina, adding that he often struggles to suggest to students that, "if you are interested in engineering, that might be a good reason to go to calculus, but if you are interested in business or

STEM Education: Opening Gateways to Learning & Careers

Editor's Note: Unlocking STEM Pathways For All Students

the humanities or social sciences, there are different paths you might go, even if you are a top-achieving math student."

On face value, new expectations for students already seem to be moving toward statistics. Both the Common Core State Standards, on which many states' math requirements are based, and the Next Generation Science Standards call for teaching data analysis and statistics, both on their own and in the process of learning other concepts.

But Kochevar warned: "There's a huge disconnect; if you look closely at the science standards, they are expecting students to have tremendous faculty with using data by middle school, but if you look at the courses, it's really not clear where those skills are supposed to be filled."

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Both sets of standards need more integration of data and statistics, he and others argue, because they were developed in the early years of the big data boom. Studies tracking data worldwide through the years **have found people produced 1.5 exabytes of new data in 1999**—or roughly 250 megabytes of data for every person alive—but by 2011, when states were adopting and implementing the math standards, **people produced more than 14 exabytes a year**. Today, people worldwide produce 2.5 exabytes of data every *day*, and the **total data have doubled every two years**.

Ironically, the rapid expansion of big data and statistics use in the broader society and economy comes at the same time American students seem to be struggling with those concepts. From 2007 to 2017, 4th and 8th students' scores on the National Assessment of Educational Progress in mathematics fell significantly on problems related to data analysis, statistics, and probability—a decline that helped drive overall dips on the math test in 2017.

In part, experts say, that's because statistics and data analysis have traditionally taken a back seat to calculus in high school math, and most students already have difficulty completing the classical path.

"The idea that statistics is hard is grounded in that fact that if you took statistics 10 years ago, you had to take calculus first, and the statistics used formal probability ... with theorems that built on calculus," said Uri Treisman, a mathematics professor and the executive director of the Charles A. Dana Center at the University of Texas at Austin. He's been working with K-12 and university systems to develop a statistics pathway as an alternative to classical calculus.

It's an idea that others have pushed back on, by situating a high school statistics pathway as either advanced material only suitable for students who have already passed calculus—or a less-rigorous path for students who can't hack it in classical math.

"Any time you have multiple pathways, the advantaged will capitalize on one and that will become the 'real' one," Treisman said. "If we are going to create data science pathways, they had better be anchored in things that lead to upward social mobility and have a rigor to them. We have to make sure new pathways have at least equal status as the traditional one—and ensure everyone has access to them. If we allow [statistics and data] to be the easy or weaker path, we relinquish the commitment to equity we started with."

Mixed Signals in Calculus

For a picture of how severe that inequity can get, one only has to look at calculus.

Until about 1980, calculus was seen as a higher education course, primarily for those interested in mathematics, physics, or other hard sciences, and only about 30,000 high school students took the

course. That began to change when school reformers glommed onto calculus as an early example of a rigorous, college-preparatory course, said David Bressoud, a mathematics professor at Macalester College and a former president of the Mathematical Association of America, who has examined the evolution of calculus studies.

"The more schools did this, the greater the expectation that they would do it" from parents, and district leaders—and in particular from colleges and universities, Bressoud said. "It's not just math majors or engineering majors; this has become an accepted requirement for admission to top universities. You are not going to get into Duke if you haven't taken calculus, even if you plan to major in French literature."

Today, some 800,000 students nationwide take calculus in high school, about 15 percent of all high schoolers, and nearly 150,000 take the course before 11th grade. Calculus classes have been and remain disproportionately white and Asian, with other student groups less likely to attend schools that offer calculus or the early prerequisites (like middle school algebra) needed to gain access to the course.

For example, in 2015-16, black students were 9 percentage points less likely than their white peers to attend a high school that offered calculus and half as likely to take the class if they attended a school that offered it. And if black students did get into a class, their teachers were also less likely to be certified to teach calculus than those of white students, according to an Education Week Research Center analysis of federal civil rights data.

And despite the rapid growth of calculus as a gold standard, university calculus experts argue it is a much weaker sign that a student is actually prepared for postsecondary math in the science fields than it appears.

In fact, **a new report by the Mathematics Association of America and the National Council of Teachers of Mathematics** found many students who took Advanced Placement Calculus AB still ended up retaking calculus in college—and 250,000 students end up needing to take even lower-level courses, like precalculus or algebra.

In the end, the report found taking calculus in high school was associated with only a 5 percentage point increase on average in calculus scores in college—from 75 percent to 80 percent. Rather, the best predictor of earning a B or better in college calculus was a student earning no less than As in high school Algebra 1 and 2 and geometry.

So if high school calculus isn't the best indicator of a student prepared for college-level math, what does it signify in college admissions? In a word: Money.

More than half of students who take calculus in high school come from families with a household income above $100,000 a year, according to **a study this month in the Journal for Research in Mathematics Education**. By contrast, only 15 percent of middle-income students and 7 percent of those in the poorest 25 percent of families take the course.

"Math is even more important to upward mobility now than it was 20 or 30 years ago, because ... it's seen as related to your general ability to solve problems quickly," Treisman said, adding that as a result, "there's general anxiety and panic about equity issues for anything new, even though the current [calculus] pathway is a burial ground for students of color."

Forging a New Path

Statistics and data literacy advocates hope diversifying the field of interesting and rigorous math courses could broaden students' path to STEM and other careers. As of 2017, the **U.S. Bureau of Labor Statistics estimations** showed that jobs that require data literacy and statistics are among the 10 fastest-growing occupations in the country.

"We have two paths forward," said William Finzer, a senior scientist at the Concord Consortium, which works with school districts to improve their math curricula. "The easier one—like **the path computer science took**—is to develop a course or a subject area and get schools to give it time. ... The problem of that is, it doesn't spread the opportunity very widely. It becomes concentrated in the small group of kids who elect to take the course—and it's just one more subject to take."

Finzer instead envisions a more holistic approach in which at least one class a year—be it math, biology, or even civics or history—asks students to grapple with making sense of large data sets. Such an approach, he said, "would make a huge difference, because it would mean when you came out of high school, data would not be foreign to you."

EDC's Oceans of Data Institute is building learning progressions for statistics and data literacy at different grades. The progression would include concepts in statistics and data literacy, but also computer science—to be able to use common programming and tools used by data professionals—and more philosophical concepts, such as the ethical use of statistics and privacy protections.

Education Week Researcher Alex Harwin contributed to this report.

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Published in Print: May 23, 2018, as **Move Over, Calculus. Statistics Is on the Rise**

Progression for Statistics and Data

EDC's Oceans of Data Institute is building learning progressions for statistics and data literacy at different grades. Randy Kochevar, who directs the institute, said they are based on the acronym **CLIP**, meaning students learn how to use:

Complex, multi-variable data ("We're not just looking at hours of sunlight and heights of bean plants," he said);

Larger data sets than students need to answer any one question, so they are forced to sort and understand relevance;

Interactively accessed data, rather than sample graphs just written out on paper; and

Professionally collected data that forces students to think about how and why it was collected—and what biases may exist in the samples.

Source: Oceans of Data Institute

MAA
MATHEMATICAL ASSOCIATION OF AMERICA

The Changing Face of Calculus: First-Semester Calculus as a High School Course

The Changing Face of Calculus: First-Semester Calculus as a High School Course

*This article will appear in the August/September issue of FOCUS
David M. Bressoud, Macalester College

Once upon a time, calculus was the first college-level mathematics course taken by mathematically talented students. The students in first-semester calculus were mathematically motivated, generally well prepared, and they were seeing these ideas for the very first time. This is no longer true. Most of our best-prepared mathematics students arrive in college with credit for at least the first semester of calculus, many of them with credit for both semesters. Despite steady growth in majors in science and engineering, enrollment in first-semester calculus has been flat or slightly declining at both two- and four-year undergraduate programs. It is the College Board?s Advanced Placement Calculus Program that has been growing steadily at 7?8% per year (see figure 1).

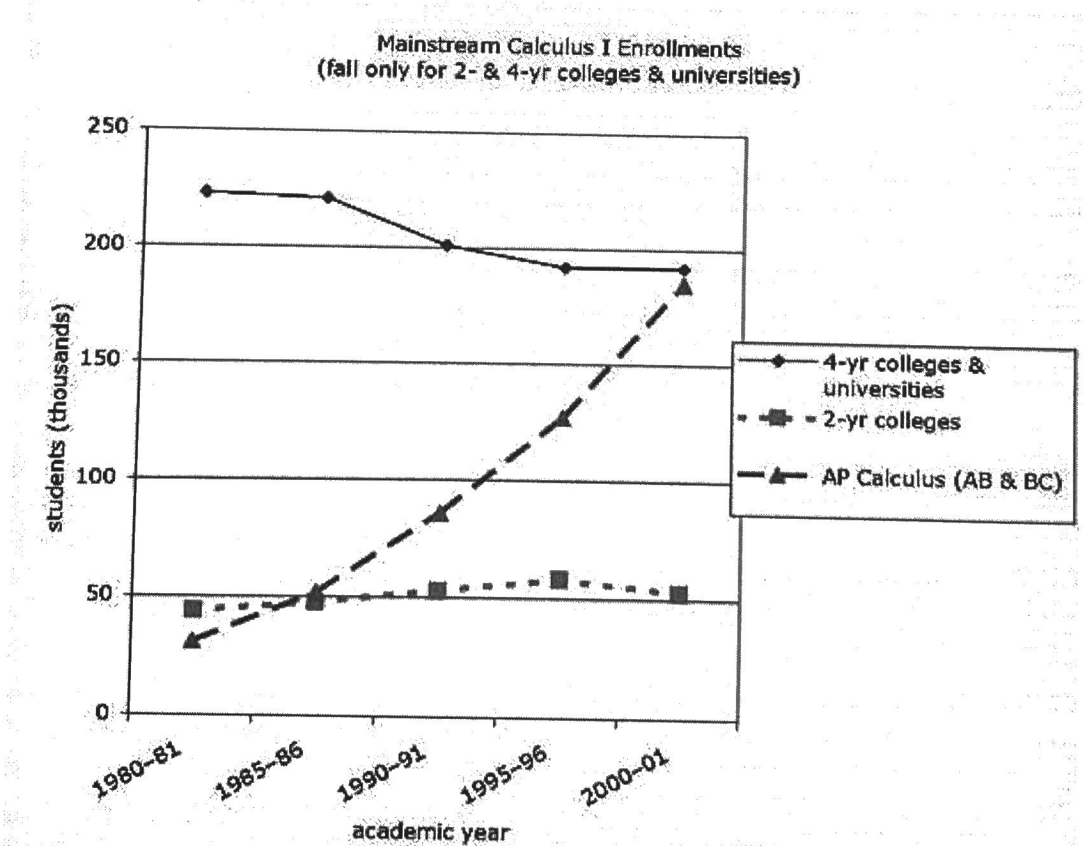

Figure 1: Mainstream Calculus I Enrollments. Fall two- and four-year college and university enrollments from [4] . AP Calculus enrollments from The College Board (most recent years available at [1]).

In 2004 over 225,000 high school students took the AP calculus exam. This number is far larger than the number of students who took mainstream first-semester calculus in all four-year undergraduate programs in the Fall of 2000. By the time of the next CBMS survey in 2005?06, we can expect that more students will take an AP Calculus exam than will take mainstream Calculus I in the Fall of 2005 in all 2-year and 4-year institutions combined.

First-semester calculus has become a high school topic for most of our strongest students. This has several implications:

1. We should ensure that students who take calculus in high school are prepared for the further study of mathematics.
2. We should address the particular needs of those students who arrive in college with credit for calculus.
3. We should recognize that the students who take first-semester calculus in college may need more support and be less likely to continue with further mathematics than those of a generation ago.

This article will address the implications for calculus taught in high schools. A second article, ?The Changing Face of Calculus: First- and Second-Semester Calculus as College Courses,? will look at the implications for how we teach calculus in colleges and universities.

Recommendations for High School Calculus

The pressure to take calculus in high school is understandable. Competition for admission to the best colleges and universities is fierce. It has helped to create strong growth in AP programs across the board. Many mathematicians deplore this movement of calculus from the college to the high school curriculum, but the pressures are too strong to stop or even substantially slow it. What we can hope is to shape it.
With this in mind, the presidents of the MAA and NCTM issued a joint statement in 1986 [3] with two strong recommendations which I paraphrase here:

1. In spite of the pressures to take calculus while still in high school, students should never short-change their mathematical preparation in subjects such as algebra, geometry, or trigonometry. Solid mathematical preparation is far more important than exposure to calculus.
2. When calculus is taught in high school it should be a college-level course. This means that the goal of the course should be to give students the same breadth of topics and mastery of calculus obtained by students taking such a course in college. It means that the course should be taught with the expectation that students who perform satisfactorily will be able to place into the succeeding college calculus course.

I believe that these recommendations need to be repeated and re-emphasized. One of the inevitable weaknesses of the AP program is that student enrollment in an AP class appears on the transcript that is reviewed for college admission, but the test that evaluates whether or not the student has learned this material at a college level is not administered until after college acceptances have been sent out. This is why many students enroll in AP courses but do not take the examinations. Many schools are under pressure to offer a course that is nominally an AP Calculus course, even if they expect few students will be able to pass the AP exam. These recommendations are intended to back up the teachers who are trying to resist rushing students into calculus before they are properly prepared.

It is particularly important that the calculus taught in high school should be a substantive course that prepares students for further work in mathematics. A weak overview of calculus does little to reinforce student knowledge of algebra, geometry, or trigonometry. In fact, it may encourage slighting these subjects in order to get into the calculus course that will improve the appearance of one?s transcript. On the other hand, a solid calculus course should require and help develop a level of mastery of these core subjects that is essential for any further work in mathematics.

Finally, these recommendations recognize that the students who take calculus in high school are among our best students. They must be prepared for college-level mathematics. Once they are ready for and are studying calculus, they should be learning it in a course that is comparable to what they would see in a mainstream college course.

The Responsibilities of Mathematicians

How calculus is taught is important. As I argued in 1992 [2], calculus is not only essential for building mathematical models of the world around us and thus informing disciplines such as physics, economics, and biology, its creation/discovery was the defining moment in the birth of modern mathematics. It has shaped our modern conception of and expectations for mathematics. Calculus should not be the only pillar supporting the undergraduate curriculum in mathematics. Discrete mathematics, geometry, and data analysis have equally important if very different roles to play. But calculus must remain one of those pillars. To ensure that it remains so, mathematicians must be concerned about how it is taught both in colleges and in high schools.

Calculus can be and is being taught well in high schools, but as the number of high school calculus courses expands, so does the number of high school teachers who must teach these courses without much more preparation than the undergraduate course they themselves took, often many years before. At many high schools, only one person teaches calculus, and so peer support may be lacking. The purpose of the AP Calculus examinations is to provide a common standard against which to measure students from all of these classes, but it can only accomplish so much. Ultimately, the way to ensure that what is taught in high school calculus really is a college-level course is through the preparation and support of the teachers who will lead these classes.

The College Board runs many workshops for AP Calculus teachers. NCTM meetings include well-attended sessions that address their needs. The MAA is beginning to realize its own potential in this area. But there still remain far too few university-level mathematicians who are willing to assist in the task of preparing and supporting high school teachers. At the very least, all mathematicians have a responsibility to be aware of the AP Calculus program: its course expectations and the nature of its examinations. Every department should encourage at least one individual to attend the annual AP Reading (the grading of the free response questions), to work with local AP Calculus teachers, or to help prepare and support those who will teach calculus in high school.

Calculus II as a High School Class

The same pressures that are pushing Calculus I into the high school curriculum are doing the same for Calculus II. Traditionally, it was a very elite group of students who took BC Calculus, covering the entire two-semester college syllabus. That group of students also grew by 6?8% per year until the mid-1990s. Over the period 1995?98, the rate of growth of BC calculus accelerated to 10?11% per year, a rate that has held up since then. In 2004, the number of students taking the BC Calculus exam exceeded 50,000. It will likely exceed 60,000 by 2005?06, the year of the next CBMS survey.

In 2002, 23% of the students who took BC Calculus did so before their senior year [7]. These high school students are not necessarily well served by taking classes in linear algebra, several variable calculus, or differential equations at a local college. Picking up additional college credits is far less useful for them than deepening and broadening the mathematics they already think they know. These students need to be challenged, but they also need to be prepared for and enticed into a deep study of further mathematics in the company of their peers.

There are many local programs that recognize this. In Minnesota, we have the University of Minnesota Talented Youth Math Program (UMTYMP). At the North Carolina School of Science and Mathematics, the mathematics department is developing courses that return to calculus, using several variables, differential equations, and modeling to explore its topics in greater depth. But not enough students have access to these kinds of programs. There is a need for a substantial national effort to create materials that can be used with these students and to help teachers learn how to use them.

The movement of calculus into the high schools is not necessarily bad, but it does require the efforts of the mathematical community'individuals, departments, and professional associations'to prepare and support those who will teach it and to resist the pressures that would weaken it.

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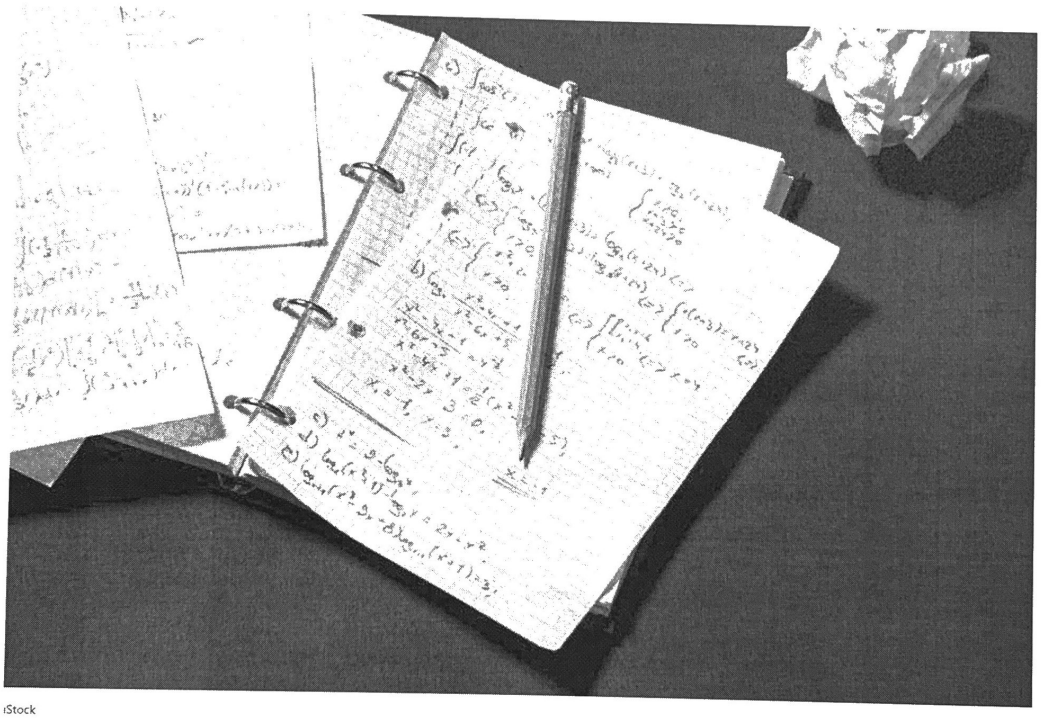
Solving the problem of the calculus whiz



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Researchers sought answers on college success in study of more than 6,000 freshmen

Peter Reuell
Harvard Staff Writer
July 27, 2018

Calculus.

The word alone is enough strike terror into the hearts of even the most accomplished students, but for those who break out in cold sweats at the thought of differentiation rules and integral tables, researchers Philip Sadler and Gerhard Sonnert bring a message of hope.

Contrary to conventional wisdom, taking high school calculus isn't necessary for success in college calculus. What's more important is mastering the prerequisites — algebra, geometry, and trigonometry — that lead to calculus. That insight comes from a study of more than 6,000 college freshmen at 133 institutions carried out by the Science Education Department of the Harvard Smithsonian Center for Astrophysics. Sadler, the Frances W. Wright Senior Lecturer on Celestial Navigation and Astronomy, and Sonnert, a research associate, led the study, which was described in a paper published in May in the Journal for Research in Mathematics Education.

"We study the transition from high school to college, and on one side of that there are college professors who say calculus is really a college subject, but on the other side there are high school teachers who say calculus is really helpful for their students, and the ones who want to be scientists and engineers get a lot out of it," Sadler said. "We wanted to see if we could settle that argument — which is more important, the math that prepares you for calculus, or a first run-through when you're in high school followed by a more serious course in college?"

The study's results, Sadler said, provide a clear answer — a firm grip on precalculus subjects had twice the impact of a high school course. And among those who took calculus in high school, it was the weakest students who got the most from the class.

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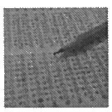

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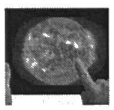

Understanding student weaknesses
Best science teachers can predict their pupils' misconceptions, study says

In designing their study, Sadler and Sonnert sought not only demographic data, but also information on students' educational history and mathematics training.

"They fill out the detailed survey at the beginning of the semester ... and there's a field on the last page where the faculty member can put their grade," Sonnert said. "Then the professors remove the first page with the student's name and we get their final grade and all the self-reported information."

"We looked at how students did in college calculus ... and tried to figure out what the predictive influence of taking a calculus course in high school was versus mastering those precalculus subjects," Sadler said.

There's no easy answer to why weaker students who take calculus in high school get the most out of it, the researchers said. The educational environment of high school calculus may explain some of the difference, Sadler said.

A high school class, he said, might have just 15 or 20 students, each receiving constant support from the teacher. In college, a student is among dozens, if not hundreds, in a lecture hall, with no opportunity for one-on-one contact with the professor except during office hours. In some cases, attending sections and even completing problem sets is optional, so unless students make an effort to seek out tutoring, it's easy to fall behind.

"Even Harvard students run into this — they have trouble with learning how to be an independent learner," Sadler said. "But one other difference is that in college the professor just assumes you know all the prerequisites, and if you don't, or you're not really solid in them, then what do you do? They won't go back and cover the things that you may be missing like a teacher can do in high school."

Ultimately, Sadler said, the study shows that success in calculus — whether in high school or college — comes more from having a strong foundation.

"The one thing the paper says is if your background is strong, if you really know your algebra, geometry, and precalculus, you're going to do well in college calculus," Sadler said. "You don't need a high school calculus course. That was a surprise. There is no reason that those new to calculus should not take the course in college, in spite of half the students in class having taken it in high school.

"There are always these kinds of arguments in education, where people have very strong views based primarily on personal experience, and we specialize in investigating those views," he said. "As it turns out, in this case the professors are more right than high school teachers, because how well students did in courses before calculus makes the biggest difference in their college calculus grade. But the heavy lifting is done by those math teachers whose efforts lay the foundation for later student success."

Editor's Picks