

Name Key

Date \_\_\_\_\_ Period \_\_\_\_\_

● e for x.

$$1. \sqrt[3]{x-10} = -24$$

$$(\sqrt[3]{x-10})^3 = (-24)^3$$

$$x-10 = -512$$

$$x = -502$$

$$2. \sqrt[4]{6x+1} = (2\sqrt[4]{3-2x})^4$$

$$6x+1 = 16(3-2x)$$

$$6x+1 = 48-32x$$

$$38x = 47$$

$$x = \frac{47}{38} = 1.24$$

$$3. \sqrt{4x-1} = 3$$

$$\sqrt{4x-1} = 4$$

$$4x-1 = 16$$

$$4x = 17$$

$$x = \frac{17}{4} = 4.25$$

Describe the transformations of the following functions from their parent graphs:

4.  $f(x) = -\frac{2}{5}\sqrt{x-2} + 1$

Reflection over the x-axis  
Vert. Comp. of  $\frac{2}{5}$   
Shifted ~~right~~ right 2  
Shifted up 1

5.  $f(x) = \sqrt[3]{2(x+1)} - 6$

Shifted down 6  
Shifted left 1  
Horizontal Comp. of  $\frac{1}{2}$

6.  $f(x) = 3\sqrt{-x} + 2$

Vertical stretch of 3  
Reflection over y-axis  
Vertical Shift up 2

Write the function given the following information:

7. The parent function  $f(x) = \sqrt{x}$  reflected across the y-axis, horizontally stretched by 3, and shifted up 1.

$$f(x) = \sqrt{-\frac{1}{3}x} + 1$$

8. The parent function  $f(x) = \sqrt[3]{x}$  vertically compressed by  $\frac{1}{2}$ , shifted right 1, and shifted down 5.

$$f(x) = \frac{1}{2}\sqrt[3]{x-1} - 5$$

List the characteristics of the following functions:

9.  $f(x) = \frac{x+3}{x^2-9} = \frac{x+3}{(x+3)(x-3)}$

Hole(s):  $x = -3$   
Zero(s): none  
Vertical Asymptote(s):  $x = 3$   
Horizontal Asymptote:  $y = 0$   
Slant Asymptote: none

10.  $f(x) = \frac{x^2-5x+6}{x-1} = \frac{(x-3)(x-2)}{x-1}$

Hole(s): None  
Zero(s):  $x = 3, x = 2$   
Vertical Asymptote(s):  $x = 1$   
Horizontal Asymptote: none  
Slant Asymptote:  $y = x - 4$

$$\frac{x^2-5x+6}{x-1} = \frac{x-4}{x-1} + \frac{-4x+6}{x-1}$$

11.  $f(x) = \frac{2x^2-32}{x^2-4}$

Hole(s): None  
Zero(s):  $x = \pm 4$

Vertical Asymptote(s):  $x = \pm 2$

Slant Asymptote: None

Horizontal Asymptote:  $y = 2$

$$12. f(x) = \frac{1}{x-5} + 3$$

Vertical Asymptote:  $x=5$

Horizontal Asymptote:  $y=3$

Domain:  $x | x \neq 5$

Range:  $y | y \neq 3$

$$13. f(x) = \frac{2}{x+1}$$

Vertical Asymptote:  $x=-1$

Horizontal Asymptote:  $y=0$

Domain:  $x | x \neq -1$

Range:  $y | y \neq 0$

Determine the inverse of each function

$$14. f(x) = 2(x-3)$$

$$x = 2(y-3)$$

$$f^{-1}(x) = \frac{x}{2} + 3$$

$$15. f(x) = \frac{1}{3}x + 6$$

$$x = \frac{1}{3}y + 6$$

$$3(x-6) = y$$

$$f^{-1}(x) = 3x - 18$$

Write the original function and solve. Round answers to 2 decimal places.

16. A new stock has just been offered claiming 15% interest per year on your investment. You decide to put in your life savings of \$12,000 into the account. How many years will it take for your money to double its value?

$$24000 = 12000 (1.15)^t$$

$$2 = 1.15^t$$

$$\log 2 = \log 1.15^t$$

$$\frac{\log 2}{\log 1.15} = \frac{t \log 1.15}{\log 1.15}$$

$$t = 4.96 \text{ years}$$

17. A new virus causes the red blood cell count in men to decrease at a rapid rate making them vulnerable to illness. If a 40 year old man has 4,500,000 red blood cells originally, how many days until he will be down to 2,000,000 red blood cells if the virus causes a decrease of 38% per day?

$$2,000,000 = 4,500,000 (1 - .38)^t$$

$$0.44 = .62^t$$

$$\frac{\log .44}{\log .62} = t$$

$$t = 1.7 \text{ days}$$

18. A bank has just celebrated its grand opening offering new customers a savings account that will yield 13% interest compounded continuously. If your parents decide to start a new account for you with \$800, how much money will be in the account after 25 years?

$$A = 800e^{(.13)25}$$

$$\$20,632.27$$

Determine the inverse of the following functions.

$$19. f(x) = \log_3 2x + 1$$

$$x = \log_3 2y + 1$$

$$x-1 = \log_3 2y$$

$$3^{x-1} = 2y$$

$$f^{-1}(x) = \frac{3^{x-1}}{2}$$

$$20. f(x) = e^{x+3} - 2$$

$$x = e^{y+3} - 2$$

$$x+2 = e^{y+3}$$

$$f^{-1}(x) = \ln(x+2) - 3$$

21.  $f(x) = 5\ln(x-2) + 10$

$x = 5\ln(y-2) + 10$

$e^{\frac{x-10}{5}} = y(y-2)$   $f'(x) = e^{\frac{x-10}{5}} + 2$

22.  $f(x) = \frac{1}{2}(4)^x$

$x = \frac{1}{2}(4)^y$

$2x = 4^y$

$\log_4 2x = y$

$f^{-1}(x) = \log_4(2x)$

Expand the following logarithms. Simplify if possible.

23.  $\log_3 \frac{27x^2}{z}$

$\log_3 27 + 2 \log_3 x - \log_3 z$

$3 + 2 \log_3 x - \log_3 z$

24.  $\ln \sqrt[3]{xy}$

$\frac{1}{3}(\ln x + \ln y)$

$\frac{1}{3} \ln x + \frac{1}{3} \ln y$

25.  $\log \frac{10z}{xy^8}$

$\log 10 + \log z - \log x - 8 \log y$

$1 + \log z - \log x - 8 \log y$

Condense the following logarithms. Simplify if possible.

26.  $\frac{1}{2} \ln x - \ln y - \ln 4$

$\ln \frac{\sqrt{x}}{4y}$

27.  $\log_4 16 + \log_4 4 - \log_4 64$

$\log_4 \frac{64}{64} = \log_4 1 = 0$

Solve the following Exponential and Logarithmic Equations.

28.  $4^{x-5} = \frac{1}{64} x^{x+1}$

$4^{x-5} = (4^{-3})^{x+1}$

$x-5 = -3x-3$

$4x = 2$

$x = \frac{1}{2}$

29.  $\log_3 4x + \ln 4 = 6$

$\log_3 24x = 6$

$3^6 = 24x$

$\frac{729}{24} = x$

$30.375$

30.  $12^{x+9} = 80$

$12^{80} = x+9$

$\frac{\log 80}{\log 12} - 9 = x$

$x = -7.24$

31.  $\log_3 x + \log_3(x+6) = 3$

$\log_3(x^2+6x) = 3$

$3^3 = x^2+6x$

$x^2+6x-27=0$

$(x+9)(x-3)=0$

~~$x = -9$~~   $x = 3$

Evaluate the following piecewise functions for the values of  $f(-3)$  &  $f(7)$ .

2.  $f(x) = \begin{cases} 2x-6 & \text{if } -6 \leq x < -2 \\ -2x^2+5 & \text{if } -2 \leq x < 12 \end{cases}$

$f(-3) = -12$

$f(7) = -93$

33.  $f(x) = \begin{cases} 12 & \text{if } -15 \leq x < 2 \\ 4x & \text{if } 2 \leq x < 5 \\ -9x-7 & \text{if } 5 \leq x \end{cases}$

$f(-3) = 12$

$f(7) = -70$

Student's heights in an 8<sup>th</sup> grade classroom are normally distributed with an average of 66 inches and a standard deviation of 3 inches. Determine the following:

34. The probability that a randomly selected student is less than 62 inches tall.

$$\frac{62-66}{3} = -\frac{4}{3} \xrightarrow{\text{use chart}} .0968$$

35. The probability that a randomly selected student is between 60 and 65 inches tall.

$$\frac{60-66}{3} = -2 \rightarrow .0228 \quad \frac{65-66}{3} = -\frac{1}{3} \rightarrow .3085$$

$$.3085 - .0228$$

$$\boxed{0.2857}$$

36. The probability that a randomly selected student is more than 70 inches tall.

$$\frac{70-66}{3} = \frac{4}{3} \rightarrow .9032 \quad 1 - .9032 = \boxed{0.0968}$$

An automobile insurance company claims that its rates for teenage drivers average \$400 less per year than the same coverage from another company. In a random sample of 60 customers, the average savings was \$395 per year with a standard deviation of \$6 per year.

37. What is the z-score of the company's claim?

$$\frac{395 - 400}{\frac{6}{\sqrt{60}}} = \frac{-5}{0.775} = -6.45$$

38. Is there enough evidence to reject the company's claim? Explain your reasoning.

Yes because  $|-6.45|$  is greater than 1.96

39. The Republican and Democratic parties are in a debate over who really won the 2014 election of a new Senator in Georgia. The Republican's claim their candidate, Harvey Handshake, gained 55% of the vote, while the Democratic candidate, Franklin Fistbump, only gained 45% of the vote. The Democrat's don't dispute the results, but claim there is no winner because of a margin of error equal to  $\pm 3\%$ . Is there a clear winner in this election? Explain your reasoning.

FF 42-48%  
HH 52-58%

Yes, because the margins of error don't overlap.

Given the following functions:  $f(x) = 2x - 8$ ,  $g(x) = x^2 - 3x - 4$ , and  $h(x) = -4x - 9$  determine the following:

40.  $(g-h)(x)$

$$(x^2 - 3x - 4) - (-4x - 9)$$

$$\boxed{x^2 + x + 5}$$

41.  $f(g(x))$

$$2(x^2 - 3x - 4) - 8$$

$$\boxed{2x^2 - 6x - 16}$$

42.  $g(h(-2))$

$$h(-2) = -1$$

$$g(-1) = 0$$

$$\boxed{0}$$

43.  $(fh)(x)$

$$(2x - 8)(-4x - 9)$$

$$\boxed{-8x^2 + 14x + 72}$$

44.  $\left(\frac{g}{f}\right)(x)$

$$\frac{x^2 - 3x - 4}{2x - 8} = \frac{(x+1)(x-4)}{2(x-4)}$$

$$\boxed{\frac{x+1}{2}}$$

45.  $g(f(x))$

$$(2x - 8)^2 - 3(2x - 8) - 4$$

$$4x^2 - 32x + 64 - 6x + 24 - 4$$

$$\boxed{4x^2 - 38x + 84}$$