## Should colleges drop calculators in math class?

Posted by B. Rose Huber-Pittsburgh
U. PITTSB URGH (US) - Math instructors promoting calculator usage in college classrooms may want to rethink their teaching strategies. That's according to Samuel King, postdoctoral student in the University of Pittsburgh's Learning Research and Development Center, who has proposed the need for further research regarding calculators' role in the classroom. King has conducted a limited study with undergraduate engineering students published in the British Journal of Educational Technology."We really can't assume that calculators are helping students," says King. "The goal is to understand the core concepts during the lecture. What we found is that use of calculators isn't necessarily helping in that regard." Together with Carol Robinson, coauthor and director of the Mathematics Education Centre at Loughborough University in England, King examined whether the inherent characteristics of the mathematics questions presented to students facilitated a deep or surface approach to learning.

Using a limited sample size, they interviewed 10 second-year undergraduate students enrolled in a competitive engineering program. The students were given a number of mathematical questions related to sine waves-a mathematical function that describes a smooth repetitive oscillation-and were allowed to use calculators to answer them. More than half of the students adopted the option of using the calculators to solve the problem. "Instead of being able to accurately represent or visualize a sine wave, these students adopted a trial-and-error method by entering values into a calculator to determine which of the four answers provided was correct," says King. "It was apparent that the students who adopted this approach had limited understanding of the concept, as none of them attempted to sketch the sine wave after they worked out one or two values."

After completing the problems, the students were interviewed about their process. A student who had used a calculator noted that she struggled with the answer because she couldn't remember the "rules" regarding sine and it was "easier" to use a calculator. In contrast, a student who did not use a calculator was asked why someone might have a problem answering this question. The student said he didn't see a reason for a problem. However, he notes that one may have trouble visualizing a sine wave if he/she is told not to use a calculator. "The limited evidence we collected about the largely procedural use of calculators as a substitute for the mathematical thinking presented indicates that there might be a need to rethink how and when calculators may be used in classes - especially at the undergraduate level," says King. "Are these tools really helping to prepare students or are the students using the tools as a way to bypass information that is difficult to understand? Our evidence suggests the latter, and we encourage more research be done in this area."

King also suggests that relevant research should be done investigating the correlation between how and why students use calculators to evaluate the types of learning approaches that students adopt toward problem solving in mathematics.

## Back to School/Do the Math: Counting too much on calculators

Some of Katie Bungo's fifth-grade students question why they should puzzle over basic math facts the old-fashioned way when punching a few calculator keys will instantly yield an answer. So the teacher at Horace Mann Elementary School in the Indiana Area School District is ready with a comparison: Calculator use is like her daily drive to school, a faster approach than walking, though it's no guarantee of success. "Driving doesn't necessarily mean I'm going to end up at my school ifI don't know what I'm doing," she tells her class.

In the same way, calculators can't replace the basic understanding of math needed to know if an answer displayed on a screen makes sense. That's why she lets her students use calculators to speed up their math, but only after they sh ow her they understand the concept behind the work. "Calculators are not magic,"she said. More than 40 years after the invention of these hand-held devices that have changed the way math is taught, educators still grapple with their proper role.

Few adv oc ate barring them from class until high school or college, said Henry Kepner, president of the National Council of Teachers of Mathematics. These days, debatecenters on howbest to use the now-ubiquitous devices so they encouragebig picture thinking, withoutletting them become a crutch. Pushing buttons runs counter to a die-hard view by some parents that math is best learned the way they weretaught -- by drill and memorization alone. Some still see calculators as inevitably de-emphasizing those traditions. But some teachers such as Ms. Bungo say keeping calculators away from students would bog them down by requiring too many routine calculations by hand. And it would put them at a disadvantage on tests, such as the Pennsylvania System of School Assess ment, which allow calculator use for some problems. "We don't use clay tablets anymore. We don't use tree bark to write on. Technology keeps advancing, and we need to acknowledge that,"said John D. Baker, a professor of mathematics education at Indiana University of Pennsylvania.

Educators say geometry and other areas of math can be introduced to students sooner because of calculators. For instance, being able to display the movement of triangles or other shapes on a graphing calculator allows motion geom etry to be more easily tackled as early as middle school, rather than high school, Dr. Kepner said. Students can focus more on spotting trends and less on worrying about mistakes in single calculations. "What does a quadratic function or equation look like? I can talk about what happens ifI change some of the sy mbols,"said Dr. Kepner, who taught middle and high school math for 12 years and is a professor of math education at the University of Wisconsin-Milwaukee. "I can talk about what happens to the graph." Imagine the tedium students would face ifthey performed compound interestcalculations without a calculator. "If they did it by hand, we'd come to a stop for several minutes while everybody did the work,"he said. Tom Reardon, a high school teacher in suburban Y oungstown, Ohio, remembers showing up in 1993 for a seminar on graphing calculators, convinced he had little use for the device. But he became awestruck watching it display graphically the distance that a batted baseball would fly, once values were inputted for the ball's velocity, its angle of ascent and wind speed.

That's when it hit him, said Mr. Reardon, who recently retired from his high school job and now leads professional development classes on calculators, some underwritten by calculator maker Tex as Instruments. 'I can't have my students not see this. This is fun," he said. Y et for all the classroom math exploration that calculators open up, do the devices help or hinder one's learning of basic addition, multiplication, division and subtraction facts? On the one hand, research "doesn't really support"the view held by detractors that calculators make students less likely to memorize those basic facts, said Robert Siegler, a Carnegie Mellon University professor and member of the National Mathematics Advisory Panel, which suggests ways to enhance American students' math skills.

Nor does the research that the panel considered bolster proponents of calculator use. "The research suggests that it neither helps nor hurts learning of basic arithmetic,"he said. "There is some evidence thatit might do a little bit of good for learning functions and problem-solving skills, but basically research stopped (around) 1986." That was largely before the emergence of graphing calculators, now commonly used in classessuch as algebra and calculus to plot graphs and tackle multiple equations simultaneously on a larger screen than a basic calculator has.

Some panel members said calculators are so common in classrooms that making research comparisons havebecome more difficult. Nevertheless, the math panel last year recommended that the nation pursue additional research on the effect calculators have on students' computation, problem-solving and conceptual understanding. Calculators have come along way since 1967 , when Tex as Instruments developed a prototype. It weighed almost three pounds, a behemoth next to some basic calculators today weighing less than an ounce.

Still, the early device with just 18 keys and four functions(addition, subtraction, multiplication and division) was revolutionary. At the time, calculations were done on tabletop models that weighed more than 50 pounds, required electricity and cranked out results on spools of paper tape. The debate about ease versus understanding that has ensued ever sincehas spread to the Internet. This spring, those who teach college math were abuzz over implications of WolframAlpha, a Web site, wolframalpha.com, and computational knowledge engine that does calculations at warp speed. Its search-engine-style question bar lets anyone ask for, and receive, instant answers to complex problems.

For instance, type in the query "What is the square root of pi?" and in seconds, a decimal approximation with 58 digits appears, above a set of equations. Some say what's importantat any grade level is structuring lessons and exams so students think about the underlying meaning, rather than giving them credit for reciting numbers spit out by a display screen. "It doesn't make any sense to ask children (on an exam) to multiply 123 times 456 ifthey have a calculator,"said Dr. Siegler. "What's the point? Y ou're not testing their knowledge of multiplication. All you're testing is whether they know how to use the calculator." Being able to estimate answers becomes more important as a safeguard against hitting a wrong button. Educators say calculators should not be allowed to become a substitute for understanding.

That can happen if a calculator "is used to guess answers, rather than solving for an answer using the relevant concepts and processes,"said Tamara Lakins, an associate professor and past chair of the math department at Allegheny College.

Dr. Siegler said a better use for calculators is on problems that can't be solved merely by the calculation itself, ones in which the real challenge is knowing what steps on the calculator to take. Mr. Reardon said calculators can even sneak some math in on someone who's not ex cited by the subject but enjoys gadgets. "They get their thumbs going,"he said. "They'll learn some math almost in spite of themselves."

## Real Math Doesn't Use Calculators

## Posted by Chad Orzel

In a discussion this week with someone who spends most of her time working with students who are struggling mightily in developmental math, I heard an argument I hadn't given much thought previously: students who have passed algebra and even pre-calc in high school frequently crash and burn when they hit our developmental math, because the high schools let them use calculators and we don't.
[...][P]art of me wonders if we're sacrificing too much on the altar of pencil and paper. It's great to be able to do addition in your head and long division on paper - yes, I know, I'm old — but is it worth flunking out huge cohorts of students because their high schools let them use calculators and we don't?
(The ellipses replace a bit where he expresses "uninformed sympathy" with professors who don't allow calculators. The cut is for length, not to distort his argument)

The reason why many math departments- including the one where I work- either do not allow calculators or greatly restrict their use is very simple: Real math doesn't use calculators. If you are going to do math or science at a college level, you need to be comfortable with problems that can not be solved by punching numbers into a calculator.

I'm trying to think of a good analogue from the humanities or social sciences. I think it's reasonably accurate to say that the notion that math at the college level involves calculators is rather like the belief that history at the college level is about memorizing the names of the kings of England, or that English at the college level is about parts of speech and metrical forms, or that economics at the college level is about learning to balance your checkbook.

The ability to do numerical operations is an important background skill for math or science, but it is not by any means the core of the discipline. Most of the time, people in the mathematical sciences are working with equations as abstract objects; the numbers are secondary.

The reason for restricting the use of calculators in introductory math and science classes is that calculators will be no help in higher math classes, and can be an active impediment to learning. The sooner you can start breaking students of their fixation on numerical manipulation and move them toward being comfortable with algebra and other more abstract representations, the better.

If that's true, why do high schools math classes allow calculators? For the same reason that high school English classes continue to have vocabulary tests-high school is about establishing the minimum level of competence needed to function as a citizen in modern society. The ability to do math with a calculator is the absolute rock-bottom minimum skill required for modern society-I think it would be good to have students come out of high school being familiar with algebra (algebra is like sunscreen), but l'll settle for them being able to do math on a calculator.

College level work is supposed to be different. It's preparing students to be able to function at more than the minimum level of competence in a subject. Accordingly, college-level classes will force students to develop the skills they would need to move up to higher level classes, or at least allow them to determine that they are not sufficiently interested in the subject to develop the necessary skills, and major in something else.

That's why introductory college math classes often restrict the use of calculators, and why I have on occasion done the same with intro physics classes. If you're going to do physics beyond the level of Giancoli, or math beyond the level of introductory calculus, you will absolutely need to be able to do abstract operations that a calculator will not help with.

Now, you can ask whether what's appropriate for introductory classes is really appropriate for remedialclasses, which is the real context. If the goal is only to bring those students up to the minimum level of competence they should've had coming out of high school, then I suppose not. But to the extent that those classes are supposed to be catching students up to the point where they're prepared to do college-level math, I think it is appropriate- it's the equivalent of having a really good high school class that gives students a leg up on their first year of math, only, you know, a year late. My own inclination is to say that classes at the college level- even remedial classes at the college level- should aim higher than the minimum acceptable level, so I'm on board with calculator bans, even at the remedial level.

## Why Johnny Can't Add Without a Calculator

By Konstantin Kakaes
When Longfellow Middle School in Falls Church, Va., recently renovated its classrooms, Vern Williams, who might be the best math teacher in the country, had to fight to keep his blackboard. The school was putting in new "interactive whiteboards" in every room, part of a broader effort to increase the use of technology in education. That might sound like a welcome change. But this effort, part of a nationwide trend, is undermining American education, particularly in mathematics and the sciences. It is beginning to do to our educational system what the transformation to industrial agriculture has done to our food system over the past half century: efficiently produce a deluge of cheap, empty calories.

I went to see Williams because he was famous when I was in middle school 20 years ago, at a different school in the same county. Longfellow's teams have been state champions for 24 of the last 29 years in MathCounts, a competition for middle schoolers. Williams was the only actual teacher on a 17-member National Mathematics Advisory Panel that reported to President Bush in 2008.

Williams doesn't just prefer his old chalkboard to the high-tech version. His kids learn from textbooks that are decades old-not because they can't afford new ones, but because Williams and a handful of his like-minded colleagues know the old ones are better. The school's parent-teacher association buys them from used bookstores because the county won't pay for them (despite the plentiful money for technology). His preferred algebra book, he says, is "in-your-face algebra. They give amazing outstanding examples. They teach the lessons."

The modern textbooks, he says, contain hundreds of extraneous, confusing, and often outright wrong examples, instead of presenting mathematical ideas in a coherent way. The examples bloat the books to thousands of pages and disrupt the logical flow of ideas. (For instance, the standard geometry book for Fairfax County, which is used in schools around the country, tries to explain what a mathematical point is by analogy to pixels on TV screens, which are not in fact point-like.) Teachers at other schools in the county have told him that they would rather use the old books, too, but their principals would kill them. Other teachers have told me the same about new technologies-they, like Williams, think the technologies are ineffectual, but lack his courage to oppose them.

According to an October 2011 report, 89 percent of high school math teachers think their students are ready for college-level mathematics. But only 26 percent of post-secondary teachers think the students are ready once they get there.

This shortfall in mathematical preparation for college-bound students has existed for a long time, but it is being exacerbated by the increased use of technology. College-level math classes almost never use graphing calculators, while high-school classes invariably do. College professors want their students to understand abstract concepts; technology advocates claim their products help teach students such abstractions, but in practice they simply don't.

Take the Promethean, one of the two interactive whiteboards the school uses. When I asked a Longfellow science teacher what she could do with the Promethean she couldn't do on the blackboard, the first thing she showed me was a music video featuring a Rube Goldberg machine. She did not intend this ironically.

The second thing she showed me was a drawing of an electric circuit in which wires connect a light bulb to a battery. When the circuit was closed, the bulb lit up. This drawing goes to the heart of the technological disconnect. Her students like it when the bulb lights up, she says, because it reminds them of a video game. But this shortcut is dangerous. Learning how to visualize-as required when an electric circuit is drawn on a blackboard-is vital for developing the ability to think abstractly. You also have to make students manipulate real circuits with real batteries, with real wires that connect them and sometimes break. Showing them a toy circuit in computer software is an unhappy middle ground between these two useful teaching exercises: You neither learn how to trouble-shoot in the real world, nor do you think clearly about how electrons work.

Math and science can be hard to learn-and that's OK. The proper job of a teacher is not to make it easy, but to guide students through the difficulty by getting them to practice and persevere. "Some of the best basketball players on Earth will stand at that foul line and shoot foul shots for hours and be bored out of their minds," says Williams. Math students, too, need to practice foul shots: adding fractions, factoring polynomials. And whether or not the students are bright, "once they buy into the idea that hard work leads to cool results," Williams says, you can work with them.

Educational researchers often present a false dichotomy between fluency and conceptual reasoning. But as in basketball, where shooting foul shots helps you learn how to take a fancier shot, computational fluency is the path to conceptual understanding. There is no way around it.

The fight between those who seek a way around hard work (a "royal road to geometry," in Euclid's famous phrase), and those who realize that earned fluency is the only road to understanding goes back millennia and became particularly acrimonious in America in the last half-century in the so-called math wars. On one side are education researchers like Constance Kamii, at the University of Alabama, who argues that teaching children to add and subtract is harmful. This camp says it has insights into the way children learn that warrant departure from traditional ways of teaching math. On the other side is the consensus of working scientists and mathematicians as well as teachers like Williams, who notes that it took very smart adults thousands of years to develop modern mathematics, so it makes sense to teach it to students rather than get them to "discover" it themselves.

What is new to this fight is the totalizing power of technology. A 2007 congressionally mandated study by the National Center for Educational Evaluation and Regional Assistance found that 16 of the best reading and mathematics learning software packages-selected by experts from 160 submissions-did not have a measurable effect on test scores. But despite this finding, the onslaught of technology in education has continued. The state of Maine was the first to buy laptops for all of its students from grades seven to 12 , spending tens of millions of dollars to do so, starting with middle schoolers in 2002 and expanding to high schools in 2009.

The nation is not far behind. Though no well-implemented study has ever found technology to be effective, many poorly designed studies have-and that questionable body of research is influencing decision-makers. Researchers with a financial stake in the success of computer software are free to design studies that are biased in favor of their products. (l'm sure this bias is, often as not, unintentional.) What is presented as peer-
reviewed research is fundamentally marketing literature: studies done by people selling the software they are evaluating.

For instance, a meta-analysis of the effectiveness of graphing calculators from Empirical Education Inc. reports a "strong effect of the technology on algebra achievement." But the meta-analysis includes results from a paper in which "no significant differences were found between the graphing-approach and traditional classes either on a final examination of traditional algebra skills or on an assessment of mathematics aptitude." In that same paper, calculators were marginally helpful on a tailor-designed test. The meta-analysis included the results of the specially made test, but not the negative results from the traditional exam.

Take this gem from researchers at SRI International. They say that standardized tests don't capture the "conceptual depth" students develop by using their software, so the "research team decided to build its own assessments"-and, of course, they did relatively well on the assessments they designed for themselves. Another example: A recent study by the Education Development Center compared students who took an online algebra 1 class with students who took nonalgebra eighth-grade math.* The online students did better than those who didn't study algebra at all (not exactly surprising). But the online students weren't compared with those who took a regular algebra class.

Despite the lack of empirical evidence, the National Council of Teachers of Mathematics takes the beneficial effects of technology as dogma. There is a simple shell game that goes on: Although there is no evidence technology has been useful in teaching kids math in the past, anyone can come up with a new product and claim that this time it is effective.

I tried using one such product, Cognitive Tutor from Carnegie Learning, which claims to be "intelligent mathematics software that adapts to meet the needs of ALL students." One problem asked me to calculate the width of a doorframe, given the frame's height and a diagonal measurement of the door. After 30 seconds' work with pen and paper, I submitted my answer: 93.7 cm . But Cognitive Tutor wouldn't accept it. It wanted me to go through an elaborate and cumbersome series of steps to get its answer: 93.723. This isn't teaching math-it's teaching how to use a particular software package. The supposed "real-world applications" don't even reflect the real world. Show me a tape measure that allows you to measure to one-hundredth of a millimeter.

Though serious empirical research fails to show any beneficial effects of technology, it also doesn't demonstrate any harm. The emphasis on technology is in part damaging because of its opportunity cost, both in effort on the part of policymakers and in terms of money. It also distracts from the real problem: teachers who don't understand enough about math or science. This has been a problem for a long time.

A report earlier this year from Michigan State University showed that K through eight teachers with no math specialization (the vast majority-more than 90 percent of K through six teachers and more than two-thirds of sixth- to eighth-grade teachers) got only half the questions right on a base-line test meant to see whether they knew the material they were supposed to be teaching.* The good news is that most teachers are aware of their own limitations: Only about 10 percent of the nonmath specialization K through eight teachers said they were "confident to teach all topics" in math.

Hung-Hsi Wu, a math professor at UC-Berkeley (and another member of Bush's math panel), has been running three-week classes for elementary and middle school teachers every summer for the last dozen years. His "students" must wrestle with deep mathematical questions that both pertain directly to simple math and are poorly understood by most teachers. Why does $(-2) x(-3)=6$ ? The answer isn't straightforward, and Wu takes several pages to give it. If you don't understand it, though, you don't really understand multiplication. But Wu has only been teaching about 25-30 teachers a summer-there is money for new technology but little for comprehensive teacher training. Meanwhile, the new technology makes it easier than ever for teachers to avoid learning their subject. Promethean, the "interactive whiteboard" company, advertises as a selling point the fact that teachers can share lesson plans online. But drawing up a lesson plan is itself educative: A teacher who plans his own lecture is forced toward mastery of the material, but one who downloads a PowerPoint presentation doesn't have to know anything beyond how to download the presentation. It is a mirage of efficiency: empty calories.

The real shortfall in math and science education can be solved not by software or gadgets but by better teachers. Programs like Wu's can make more teachers more like Williams. That's where efforts should be focused, not on imagined technological solutions, which obscure more than they reveal.

In this, the new Common Core standards for math, which were adopted with lightening speed by 45 states and Washington, D.C., fall short. They fetishize "data analysis" without giving students a sufficient grounding to meaningfully analyze data. Though not as wishy-washy as they might have been, they are of a piece with the runaway adaption of technology: The new is given preference over the rigorous.

Computer technology, while great for many things, is just not much good for teaching, yet. Paradoxically, using technology can inhibit understanding how it works. If you learn how to multiply 37 by 41 using a calculator, you only understand the black box. You'll never learn how to build a better calculator that way. Maybe one day software will be smart enough to be useful, but that day won't be any time soon, for two reasons. The first is that education, especially of children, is as much an emotional process as an imparting of knowledge-there is no technological substitute for a teacher who cares. The second is that education is poorly structured.
Technology is bad at dealing with poorly structured concepts. One question leads to another leads to another, and the rigid structure of computer software has no way of dealing with this. Software is especially bad for smart kids, who are held back by its inflexibility.

John Dewey, the father of American education reform, defined miseducative experiences as those that have "the effect of arresting or distorting the growth of further experience." "Growth," he wrote, "depends upon the presence of difficulty to be overcome by the exercise of intelligence." The widespread use of computer technology is inimical to the exercise of intelligence. I fear this is no more than shouting into the wind, but resist it while you can, because once it gets locked in-as our food system is, to monocultures and antibiotics in factory farms-it will be even tougher to get away from.

## Rethinking How We Use Calculators

## By José Vilson

If ever you come across a set of math teachers, whether at a common planning meeting or a bar during happy hour, bring up the conversation of calculators and watch the sparks fly. The arguments for and against calculators have the spirited vigor of a Red Sox vs. Yankees game without the animus. One side argues for the use of efficient and available technology in the classroom, while the other argues for numeracy and fluency to the highest order.

In other words, are you old school or new school?

## Two Schools of Thought

The old school suggests that, in order to develop a rich sense of numbers and fluency, we shouldn't allow students to use calculators. In a world over-equipped with technology tools, students must be able to do operations without the calculator there. In this school of thought, calculators strip students of curiosity about how numbers work because they can arrive at the answer just by pressing a few buttons, not by going through the long-established procedure of finding the answer. Calculators already spit out everything from long multiplication and division to graphs and solutions to simultaneous equations. The old school crowd isn't completely anti-technology; many of them stand by The Geometer's Sketchpad. They just wouldn't want all the mysteries and intricacies of math unlocked so quickly.

The new school suggests that we take a different outlook on the calculator issue. If we can so readily solve problems with a calculator, then why not give one to our students? The old way of writing out the multiplication
table in both a table and list form is antiquated and tiresome. The calculator is also much more efficient, reducing the amount of time one spends on a problem. For instance, imagine trying to divide a seven digit number by a two digit number higher than 12 ? Such a task seems tedious when a calculator can do it in a fraction of the time. Even operations with fractions can be simplified with calculators, so finding things like the least common denominator or remainders feels pointless if the screen just told the student the answer.

My response to this one lies somewhere in the middle. I'm not totally against calculators. I use them frequently enough when creating answer keys for my exams -- after I take time to do the problems myself. I use them when working on my taxes, and l've used one when trying to get a new couch for my mom to fit in her apartment. (Thank you, Pythagorean Theorem!)

## The Right Tool Used the RightWay

Where I start siding with the old school mathematicians is in this: how do we knowthat the calculator is telling us the truth? Numbers don't lie, but humans make meaning of these numbers and hope to ascertain how they apply in the context given. If we rely solely on calculators without giving much thought to the number we've put down, or simply assuming the calculator is always right, then we end up with everything from wrong answers to the financial collapse of 2008.
Calculators are tools to help solve problems, not the solver of the problem itself. Our students need to develop a sense of numeracy that allows them to estimate the distance between two items, for example. We can't underestimate the usefulness of looking at numbers and making quick calculations by comparing those numbers without having to pull out a smartphone.

The calculations we make on the fly matter more than the ones we make in math or science class, yet these classes are where students get the most explicit reasons for using them. In larger math problems, I can see the usefulness of a calculator. For instance, when finding the differences between planets in scientific notation, we shouldn't have to plug away at doing operations to the first factors. However, if we insert the notation wrong in our calculator, we could end up with a number bigger than we bargained for.

We need to think about the way we use calculators, or any piece of technology. Assuming that the tool is doing exactly what we're asking it to do -- or actually has the answer to the question we've asked -- is a dangerous proposition for the non-thinker. We need a healthy balance of working within the number system and doing more complex problems. We need to let calculators serve their purpose in moderation.

